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Aeroacoustic Control of Fan Tone Noise

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Mathematical formulations of the unsteady three-dimensional flowfield induced by a rotating annular cascade interacting with an oncoming periodic gust and disturbances from an oscillating actuator surface composed as a part of the duct wall are presented on the basis of a linearized unsteady lifting surface theory. The problem of suppressing the tone noise due to interaction of the rotor with the gust by means of the actuator motion is studied. Theoretical analysis with numerical calculations is conducted for a simple harmonic sinusoidal gust, and a simple harmonic sinusoidal circumferential wave form of the actuator motion and optimum conditions of the actuator motion are investigated. There are substantial differences in the optimum phase and amplitude of the actuator motion between the conditions of suppressing the upstream and downstream acoustic powers. In the case of multiple cut-on duct modes, the actuator motions of the cut-off circumferential wave numbers are desirable to effectively suppress the total acoustic powers.

I. Introduction

SUPPRESSION of tone noise because of aerodynamic interaction between rotor blades and stator vanes or between rotor blades and inlet distortions is one of the important problems of aeroengine design. In the case of transonic fan engines with supersonic tip relative Mach numbers, the so-called buzz saw noise will be most significant, but this paper deals with subsonic rotors, from which no buzz saw noise is generated. To address this problem, many studies have been carried out [1–6], but most of the studies conducted so far are concerned with pursuing analytical methods to predict noise and with seeking the cascade configurations to avoid cut-on mode generation. It is, however, very difficult to completely suppress generation of all cut-on duct modes in the whole range of operations.

The active control is one of the adaptive methods to cope with the difficulty. In particular, application of the antisound technology to this problem is quite an attractive idea. There have been some studies conducted on the application of the antisound concept. Kousen and Verdon [7] conducted a numerical study, but the model is rather unrealistic because they dealt with a two-dimensional cascade of blades with actuators on the blade surfaces. On the other hand, the experiments by Thomas et al. [8] were not necessarily successful because of the generation of multiple radial modes. Schulten [9] theoretically studied the feasibility of controlled vibration of stator vanes as antisound sources. The success of the active control with antisound is dependent not only on understanding the construction of the acoustic field composed of the primary sounds and the secondary sounds generated as antisounds, but also on the control system in situ to optimize the secondary sound sources as described in [10,11].

What the authors are concerned with are the details of the acoustic field constructed by the primary noise and the antisounds. With this in mind, the authors would like to point out that the previous analytical studies overlook the interaction of the antisound waves with the rotor blades and/or stator vanes. This paper gives a theoretical study to investigate the effect of the interaction of the antisound waves with the rotor blades on the whole sound field and to show that taking into account this effect is crucial to the optimum control of antisound.

One of the authors has already conducted theoretical studies on the acoustic control of cascade flutter based on the antisound concept [12]. This paper applies the same concept to the problem of suppressing the noise generation because of the interaction of rotor blades with oncoming periodic gustlike wakes from cascade blades at an upstream station.

In the following sections, the outline of the mathematical formulation and solution procedure is given for the prediction of the unsteady blade loading and the disturbance flowfields induced by interaction of rotor blades with oncoming periodic gusts as well as with sound waves generated from the duct wall actuator surface. Then feasibility studies have been conducted with numerical examples, and some new, interesting findings have been obtained for desirable actuator motions to suppress multiple radial modes.

II. Description of the Model and Mathematical Formulations

Here we consider an annular cascade rotating in a duct of infinite axial extent as shown in Fig. 1. The undisturbed flow is of a uniform axial velocity W_a^*, a uniform fluid density ρ_0^*, and a uniform speed of sound a_0^*. As an antisound generator, a duct wall actuator surface that can be forced to vibrate is considered.

Under the assumption of linearization, the unsteady flowfield is composed of four components: the unsteady velocity field of the gust (G), the disturbance field because of interaction of the rotor blades with the gust (the primary noise) (BG), the acoustic disturbance field generated from the duct wall actuator surface motion (the direct secondary sounds) (W), and the disturbance field because of interaction of the rotor blades with the sound waves generated from the actuator surface (the blade-interfering secondary sounds) (BW). Note that all the four components are assumed to be of the first order small quantity. Here special notice should be paid to existence of the component (BW). Our purpose is to find the optimum geometry and motion of the actuator surface to minimize the sound power of the total acoustic field $(W) + (BG) + (BW)$.

A. Interaction of Rotor Blades with Oncoming Gust

Suppose that the oncoming gust is generated as wakes from struts or stator vanes at upstream stations, and it is composed of axial

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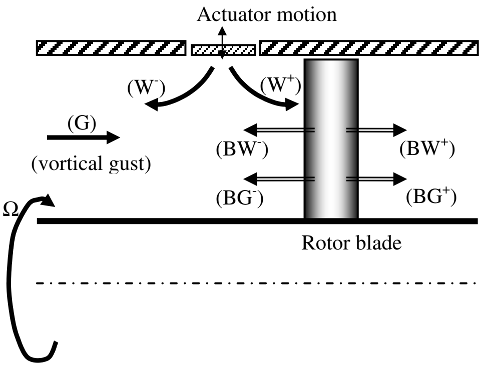

Fig. 1 A rotating annular cascade with an actuator surface on the duct wall and the components of the disturbance flowfield in the duct.

velocity disturbances only. The distorted flow can be resolved into Fourier components of a fundamental circumferential wave number equal to the number of stator vanes or struts N_G and its integer multiples. Here, however, we neglect the higher harmonics and consider a circumferentially sinusoidal distortion of the fundamental wave number N_G. Let us denote the cylindrical coordinate system fixed to the duct by $(\hat{r}, \hat{\theta}, \hat{z})$, and that fixed to the rotor by (r, θ, z), respectively. Then the axial velocity disturbance can be expressed by

$$q_{G,z}(r, \hat{\theta}, z) = -Q_G f_{N_G}(r, z) e^{iN_G\hat{\theta}} \tag{1}$$

Here Q_G stands for the amplitude of the velocity distortion normalized by the mean axial velocity W_a^*.

Let Ω be the angular velocity of the rotor rotation. Then using the relation of

$$\hat{\theta} = \theta - \Omega t, \qquad \hat{r} = r, \qquad \hat{z} = z \tag{2}$$

one can rewrite Eq. (1) in terms of the rotor-fixed coordinate system as follows:

$$q_{G,z}(r, \theta - \Omega t, z) = \tilde{q}_{G,z}(r, \theta, z) e^{-iN_G\Omega t} = -Q_G f_{N_G}(r, z) e^{-iN_G\Omega t + iN_G\theta} \tag{3}$$

As it indicates, the distorted axial flow is observed in the frame of reference rotating with the rotor as a sinusoidal gust spinning at the angular speed Ω. Then it is obvious that the unsteady blade loading because of interaction with the gust fluctuates at a frequency $N_G\Omega$, and the interblade phase angle is equal to $2\pi N_G/N_B$. Here N_B denotes the number of blades of the rotor. Therefore, the unsteady pressure difference across the mth blade surface can be expressed as

$$Q_G \Delta\tilde{p}_{BG}(r, z) e^{-iN_G\Omega t + i2\pi m N_G/N_B} \tag{4}$$

Then the disturbance pressure field $p_{BG}^{\sim}(r, \eta, z)e^{-iN_G\Omega t}$, and the disturbance velocity field $\tilde{\boldsymbol{q}}_{BG}(r, \eta, z)e^{-iN_G\Omega t}$ induced by the unsteady blade loading can be expressed by

$$\tilde{p}_{BG}(r, \eta, z) = Q_G \int_h^1 \mathrm{d}\rho \int_{-C_a/2}^{C_a/2} \Delta\tilde{p}_{BG}(\rho, \zeta) K_p(r, \eta, z - \zeta | \rho; N_B, \Omega, -N_G\Omega, N_G)\, \mathrm{d}\zeta \tag{5}$$

and

$$\tilde{q}_{BG}(r, \eta, z) = Q_G \int_h^1 \mathrm{d}\rho \int_{-C_a/2}^{C_a/2} \Delta\tilde{p}_{BG}(\rho, \zeta) \boldsymbol{K}_q(r, \eta, z - \zeta | \rho; N_B, \Omega, -N_G\Omega, N_G)\, \mathrm{d}\zeta \tag{6}$$

respectively, where $\eta = \theta - \Omega z$ denotes a helical coordinate. Here it is assumed that the rotor blades are located between $z = -C_a/2$ and $z = C_a/2$ in the annular duct with hub-to-tip ratio h, and the blades are of constant axial chord length C_a with no sweep.

The kernel functions

$$K_p(r, \eta, z | \rho; N_B, \Omega, \omega, \sigma) e^{i\omega t}$$

and

$$\boldsymbol{K}_q(r, \eta, z | \rho; N_B, \Omega, \omega, \sigma) e^{i\omega t}$$

stand for the pressure field and the velocity field induced by an annular row of pressure dipoles rotating at a rotational velocity Ω. The number of the dipoles is N_B. The dipole axes are oriented normal to $\eta = \text{const}$ surface. The strength of the dipoles is fluctuating at a frequency ω with unit amplitude and a phase angle $2\pi\sigma/N_B$ between a dipole and the next. Expressions of the kernel functions are omitted to save space, but equivalent expressions of kernel functions can be found in [13,14]. We should note here that the disturbance field is composed of an infinite number of duct modes with circumferential wave numbers $n = \nu N_B + N_G$; $\nu = 0, \pm 1, \pm 2, \ldots$.

The unsteady loading function $\Delta\tilde{p}_{BG}(\rho, \zeta)$ can be determined by the flow tangency condition on a blade surface $\eta = \theta - \Omega z = 0$, which can be written as

$$\tilde{q}_{,\perp BG}(r, 0, z) - \tilde{q}_{G,z}(r, \Omega z, z)\Omega r/\sqrt{1 + \Omega^2 r^2} = 0 \tag{7}$$

Here the symbol $\perp$ as subscript denotes the component normal to blade surfaces ($\eta = \text{const}$). From Eqs. (3), (6), and (7), one obtains an integral equation for $\Delta\tilde{p}_{BG}(\rho, \zeta)$ as follows:

$$\int_h^1 \mathrm{d}\rho \int_{-C_a/2}^{C_{a/2}} \Delta\tilde{p}_{BG}(\rho, \zeta) K_{q,\perp}(r, 0, z - \zeta | \rho; N_B, \Omega, -N_G\Omega, N_G)\, \mathrm{d}\zeta = -(\Omega r/\sqrt{1 + \Omega^2 r^2}) f_{N_G}(r, z) e^{iN_G\Omega z} \tag{8}$$

B. Sound Waves Generated from Duct Wall Actuator Surface

Let the actuator surface be located between $z = z_c - L$ and $z = z_c + L$ on the outer wall $r = 1$. We can suppose an array of loudspeakers as the actuator. We should note that the role of the actuator is to generate sound waves to suppress those associated with the disturbance flows described by Eqs. (5) and (6). To this end, the actuator should make oscillations at the same frequency as the disturbance flows to be suppressed. Therefore, let the normal displacement of the actuator surface be denoted by $a(\theta, z)e^{-iN_G\Omega t}$ in terms of the rotor-fixed coordinate system (r, θ, z). In general, the circumferential variation of the actuator surface displacement will be composed of multiple Fourier components, but for simplicity we take only a single dominant component with a circumferentially traveling wave form of a wave number n^* and a z-wise variation of the form described by $\sin\{kZ(z)\}$ with an integer k:

$$a(\theta, z)e^{-iN_G\Omega t} = A_{k\nu^*} \sin\{kZ(z)\} e^{in^*\theta - iN_G\Omega t} \tag{9}$$

where $Z(z)$ is assumed in the form

$$Z(z) = \cos^{-1}\{(-z + z_c)/L\} \tag{10}$$

and $A_{k\nu^*}$ denotes the amplitude of the actuator surface displacement divided by the duct radius r_T^*.

Note that in the present problem n^* should be an integral number given as

$$n^* = \nu^* N_B + N_G \tag{11}$$

where ν^* is an arbitrary integral number.

The actuator surface displacement in terms of the absolute coordinate system $(r, \hat{\theta}, z)$ is expressed as

$$a(\hat{\theta} + \Omega t, z)e^{-iN_G\Omega t} = A_{k\nu^*} \sin\{kZ(z)\} e^{i(\nu^* N_B + N_G)\hat{\theta} + i\nu^* N_B \Omega t} \tag{12}$$

We should note that there are an infinite number of options in choosing the driving frequency $\nu^* N_B \Omega$ ($\nu^* = \pm 1, \pm 2, \ldots$) to generate sound waves of a frequency $-N_G\Omega$ with respect to the rotor-fixed coordinate system.

As shown in [12], the oscillating actuator surface can be represented by equivalent unsteady surface mass sources. Therefore, the disturbance pressure $\tilde{p}_W e^{-iN_G\Omega t}$ generated by the actuator motion is expressed as the pressure induced by the unsteady mass sources. Applying this concept to the present model, one can ultimately obtain the following expression:

$$\tilde{p}_W(r,\eta,z) = A_{k\nu^*}\sum_{\ell=0}^{\infty} FP_W(\nu^*,\ell;z)R_\ell^{(n^*)}(r)\exp\Big[in^*\theta + i\nu^* N_B\Omega\Big(M_a^2/\beta_a^2\Big)z - \Lambda_\ell^{(n^*)}|z|\Big] \tag{13}$$

where

$$FP_W(\nu^*,\ell;z) = -\frac{R_\ell^{(n^*)}(1)}{2\beta_a^2\Lambda_\ell^{(n^*)}}\int_{z_c-L}^{z_c+L} DZ_\ell^{(n^*)}\left[\left(i\nu^* N_B\Omega + \frac{\partial}{\partial\zeta}\right)\sin\{kZ(\zeta)\}\right] \times \exp\Big[-i\nu^* N_B\Omega\Big(M_a^2/\beta_a^2\Big)\zeta - \Lambda_\ell^{(n^*)}(|z-\zeta|-|z|)\Big]\,d\zeta \tag{14}$$

$$DZ_\ell^{(n^*)} = i\nu^* N_B\Omega/\beta_a^2 - \Lambda_\ell^{(n^*)}\mathrm{sgn}(z-\zeta) \tag{15}$$

$$\Lambda_\ell^{(n^*)} = \left\{\begin{array}{ll}\sqrt{B} & : B \ge 0\\ i\,\mathrm{sgn}(\nu^* N_B\Omega)\sqrt{-B} & : B < 0\end{array}\right\} \tag{16}$$

$$B = \left\{\left(k_\ell^{(n^*)}\right)^2 - (\nu^* N_B\Omega)^2 M_a^2/\beta_a^2\right\}/\beta_a^2 \tag{17}$$

$$M_a = W_a^*/a_0^*\ (\text{axial Mach number}), \qquad \beta_a = \sqrt{1-M_a^2} \tag{18}$$

Here $R_\ell^{(n)}(r)$ and $k_\ell^{(n)}$ are normalized radial eigenfunctions and eigenvalues, respectively [13,14].

Note that $FP_W(\nu^*,\ell;z)$ denotes the normalized modal pressure amplitude, and it is independent of z in the region away from the actuator section (i.e., at $z > z_c + L$ or $z < z_c - L$).

The duct modes are identified by the circumferential wave number n^* and the radial node number ℓ, and the modes of $B > 0$ or $B < 0$ are cut-off or cut-on modes, respectively.

C. Interaction of Rotor Blades with Sound Waves from Duct Wall Actuator Surface

As already noted, the acoustic waves generated from the duct wall actuator surface interact with blades. This interaction induces another component of unsteady blade loading in addition to that described by Eq. (4). It is obvious that this component also fluctuates at the frequency $N_G\Omega$, and the interblade phase angle is also equal to $2\pi N_G/N_B$. Therefore, the unsteady blade loading on the mth blade induced by this interaction can be denoted by

$$A_{k\nu^*}\Delta\tilde{p}_{BW}(r,z)e^{-iN_G\Omega t + i2\pi N_G m/N_B}$$

Then disturbances induced by this blade loading can be written in the same manner as Eqs. (5) and (6):

$$\tilde{p}_{BW}(r,\eta,z)e^{-iN_G\Omega t} = e^{-iN_G\Omega t}A_{k\nu^*}\int_h^1 d\rho\int_{-C_a/2}^{C_a/2}\Delta\tilde{p}_{BW}(\rho,\zeta)K_p(r,\eta,z-\zeta|\rho;N_B,\Omega,-N_G\Omega,N_G)\,d\zeta \tag{19}$$

$$\tilde{\boldsymbol{q}}_{BW}(r,\eta,z)e^{-iN_G\Omega t} = e^{-iN_G\Omega t}A_{k\nu^*}\int_h^1 d\rho\int_{-C_a/2}^{C_a/2}\Delta\tilde{p}_{BW}(\rho,\zeta)\boldsymbol{K}_q(r,\eta,z-\zeta|\rho;N_B,\Omega,-N_G\Omega,N_G)\,d\zeta \tag{20}$$

The unsteady loading function $\Delta\tilde{p}_{BW}(\rho,\zeta)$ can be determined by the flow tangency condition on a blade surface, which can be written as

$$\tilde{q}_{BW,\perp}(r,0,z) + \tilde{q}_{W,\perp}(r,\Omega z,z) = 0 \tag{21}$$

It gives an integral equation for $\Delta\tilde{p}_{BW}(\rho,\zeta)$ as follows:

$$\int_h^1 d\rho\int_{-C_a/2}^{C_a/2}\Delta\tilde{p}_{BW}(\rho,\zeta)K_{q,\perp}(r,0,z-\zeta|\rho;N_B,\Omega,-N_G\Omega,N_G) = -\sum_{\ell=0}^{\infty} QWN_\ell^{(n^*)}(r,z)R_\ell^{(n^*)}(r)e^{in^*\Omega z/\beta_a^2 - iN_G\Omega(M_a^2/\beta_a^2)z - \Lambda_\ell^{(n^*)}|z|} \tag{22}$$

The expression of $QWN_\ell^{(n^*)}(r,z)$ is omitted to save space.

D. Total Disturbance Flowfield

1. Total Unsteady Blade Loading

The unsteady blade loading $\Delta\tilde{p}_B(r,z)$ is composed of $A_{k\nu^*}\Delta\tilde{p}_B(r,z)$ [solution of Eq. (22)] and $Q_G\Delta\tilde{p}_{BG}(r,z)$ [solution of Eq. (8)]. That is

$$\Delta\tilde{p}_B(r,z) = A_{k\nu^*}\Delta\tilde{p}_{BW}(r,z) + Q_G\Delta\tilde{p}_{BG}(r,z) \tag{23}$$

2. Total Acoustic Field

In the present case, the acoustic field is composed of three components: 1) the primary noise (BG) because of the interaction of blades with the gust, 2) the direct secondary sounds (W), generated from the actuator surface, and 3) the blade-interfering secondary sounds (BW) because of the interaction of blades with the sound waves from the actuator surface. We should note that the last two components (BG and BW) are generated from blades. Therefore, the acoustic pressure p is expressed as

$$p = p_{BG} + p_W + p_{BW} \tag{24}$$

The pressure p_W of the direct secondary sound waves generated from the actuator surface is already given by Eq. (13). Hereafter, the modal pressure amplitude $FP_W(\nu^*,\ell;z)$ for $z > z_c + L$ or $z < z_c - L$ is expressed as $FP_{W+}(\nu^*,\ell)$ or $FP_{W-}(\nu^*,\ell)$, respectively.

Note that p_W is of a simple harmonic time dependence of the frequency $n^*\Omega - N_G\Omega = \nu^* N_B\Omega$ and of a single circumferential mode with the wave number $n^* = \nu^* N_B + N_G$.

The expression of the blade-interfering secondary sound pressure field generated from the rotor blades because of the rotor–actuator acoustic interaction is given by Eq. (19). Rewriting it in terms of the absolute coordinate system, we finally obtain the expression of p_{BW} in the following form:

$$p_{BW} = \tilde{p}_{BW}(r,\theta,z)e^{-iN_G\Omega t} = \tilde{p}_{BW}(r,\hat{\theta}+\Omega t,z)e^{-iN_G\Omega t} = A_{k\nu^*}\sum_{\nu=-\infty}^{\infty}\sum_{\ell=0}^{\infty} FP_{BW\pm}(\nu,\ell)R_\ell^{(n)}(r)e^{i\nu N_B\Omega t + in\hat{\theta} + i\nu N_B(M_a^2/\beta_a^2)z \mp \Lambda_\ell^{(n)}z} \tag{25}$$

where $FP_{BW\pm}(\nu,\ell)$ is given by

$$FP_{BW\pm}(\nu,\ell) = -\left\{N_B/\left(4\pi\beta_a^2\right)\right\} \times \int_h^1 \rho R_\ell^{(n)}(\rho) \int_{-C_a/2}^{C_a/2} \Delta\tilde{p}_{BW}(\rho,\zeta)\left[\left(i/\Lambda_\ell^{(n)}\right) \times \left(\nu N_B \Omega^2 M_a^2/\beta_a^2 - n/\rho^2\right) \mp \Omega\right] e^{-\{i(-N_G\Omega M_a^2+n\Omega)/\beta_a^2 \mp \Lambda_\ell^{(n)}\}\zeta}\,\mathrm{d}\zeta\,\mathrm{d}\rho \tag{26}$$

Note that p_{BW} is neither of a simple harmonic time dependence nor of a single circumferential mode, but it is of multiple frequencies;

$$\nu N_B \Omega; \qquad \nu = \pm 1, \pm 2, \ldots \tag{27}$$

and of multiple circumferential modes of wave numbers

$$n = \nu N_B + N_G : \nu = \pm 1, \pm 2, \ldots \tag{28}$$

In the same manner as previously stated, the primary disturbance pressure p_{BG} due to the rotor–gust interaction is expressed by

$$p_{BG} = \tilde{p}_{BG}(r,\theta,z)e^{-iN_G\Omega t} = \tilde{p}_{BG}(r,\hat{\theta}+\Omega t, z)e^{-iN_G\Omega t} = Q_G \sum_{\nu=-\infty}^{\infty} \sum_{\ell=0}^{\infty} FP_{BG\pm}(\nu,\ell) R_\ell^{(n)}(r) e^{i\nu N_B\Omega t + in\hat{\theta} + i\nu N_B(M_a^2/\beta_a^2)z \mp \Lambda_\ell^{(n)} z} \tag{29}$$

The expression of $FP_{BG\pm}(\nu,\ell)$ is obtained by replacing $\Delta\tilde{p}_{BW}(\rho,\zeta)$ in Eq. (26) by $\Delta\tilde{p}_{BG}(\rho,\zeta)$. Note again that p_{BG} is also of multiple frequencies and of multiple circumferential modes.

3. Acoustic Power

The most suitable measure of the sound energy is the axial acoustic power. The axial acoustic enthalpy flux (i.e., the axial component of the sound intensity, normalized by $\rho_0^* W_a^{*3}$), is given by

$$I_z = \tfrac{1}{2}\left\{\left(1+M_a^2\right)\tfrac{1}{2}(\tilde{p}\bar{\tilde{q}}_z + \bar{\tilde{p}}\tilde{q}_z) + M_a^2\tilde{p}\,\bar{\tilde{p}} + \tilde{q}_z\bar{\tilde{q}}_z\right\} \tag{30}$$

Here, $\tilde{q}_z$ denotes the axial disturbance velocity, and the overline denotes the complex conjugate.

Then the axial acoustic power, normalized by $\rho_0^* W_a^{*3} r_T^{*2}$, is given by

$$E_\pm = \pm\int_h^1 r \int_0^{2\pi} I_z\,\mathrm{d}\theta\,\mathrm{d}r = \sum_{\nu(\text{cut-on})}\sum_{\ell(\text{cut-on})} E_\pm(\nu,\ell) \tag{31}$$

where $E_\pm(\nu,\ell)$ denotes the modal acoustic power, which is given by

$$E_\pm(\nu,\ell) = S_{\ell\pm}^{(n)}|FP_\pm(\nu,\ell)|^2 \tag{32}$$

Here,

$$S_{\ell\pm}^{(n)} = \pi\beta_a^2\left|\nu N_B\Omega\Lambda_\ell^{(n)}\right| / \left\{|\nu N_B\Omega|/\beta_a^2 \mp \left|\Lambda_\ell^{(n)}\right|\right\}^2 \tag{33}$$

and $FP_\pm(\nu,\ell)$ is the modal pressure amplitude given by

$$FP_\pm(\nu,\ell) = Q_G FP_{BG\pm}(\nu,\ell) + A_{k\nu^*}\{FP_{W\pm}(\nu^*,\ell)\delta_{\nu\nu^*} + FP_{BW\pm}(\nu,\ell)\} \tag{34}$$

and $\delta_{\nu\nu^*}$ denotes Kronecker delta. Further (cut on) under $\sum$ implies the sum of the cut-on modes only.

Again, note that the component of wave number $n = \nu N_B + N_G$ corresponds to the component of frequency $n\Omega - N_G\Omega = \nu N_B\Omega$.

E. Minimization of Acoustic Power

1. Minimization of Total Acoustic Power

The problem of minimizing the total acoustic power is to find the complex values of $A_{k\nu^*}/Q_G$ that minimize $E_\pm$, given by Eq. (31). We can rewrite Eq. (31) as

$$\frac{E_\pm}{Q_G^2} = A_\pm\frac{A_{k\nu^*}}{Q_G}\frac{\overline{A}_{k\nu^*}}{Q_G} + \overline{B}_\pm\frac{A_{k\nu^*}}{Q_G} + B_\pm\frac{\overline{A}_{k\nu^*}}{Q_G} + C_\pm \tag{35}$$

where

$$A_\pm = \sum_{\nu(\text{cut-on})}\sum_{\ell(\text{cut-on})} S_{\ell\pm}^{(n)}|FP_{W\pm}(\nu^*,\ell)\delta_{\nu\nu^*} + FP_{BW\pm}(\nu,\ell)|^2 \tag{36}$$

$$B_\pm = \sum_{\nu(\text{cut-on})}\sum_{\ell(\text{cut-on})} S_{\ell\pm}^{(n)} FP_{BG\pm}(\nu,\ell)\overline{\{FP_{W\pm}(\nu^*,\ell)\delta_{\nu\nu^*} + FP_{BW\pm}(\nu,\ell)\}} \tag{37}$$

$$C_\pm = \sum_{\nu(\text{cut-on})}\sum_{\ell(\text{cut-on})} S_{\ell\pm}^{(n)}|FP_{BG\pm}(\nu,\ell)|^2 \tag{38}$$

and, again, an overline denotes a complex conjugate.

Then $E_\pm/Q_G^2$ is minimum when

$$A_{k\nu^*}/Q_G = -B_\pm/A_\pm \tag{39}$$

and the minimum value is given by

$$[E_\pm]_{\min}/Q_G^2 = C_\pm - B_\pm\overline{B}_\pm/A_\pm \tag{40}$$

2. Minimization of the Acoustic Power of a Selected Frequency

Note that the summations with respect to ν and ℓ in Eqs. (36–38) are arbitrary, and the problem can be reduced to the suppression of selected acoustic modes by restricting the summations to those of the selected values of ν and ℓ. In particular, the description of the problem to minimize the acoustic power of a circumferential wave number of $n = \nu N_B + N_G$ can be obtained by omitting the summation symbol $\sum_{\nu(\text{cut-on})}$ in Eqs. (36–38). Note again that the modes of a circumferential wave number $n = \nu N_B + N_G$ correspond to a component of frequency $\nu N_B \Omega$.

3. Elimination of a Single Acoustic Mode

Omitting the summation symbols $\sum_{\nu(\text{cut-on})}\sum_{\ell(\text{cut-on})}$ in Eqs. (36–38), one obtains the solution to the problem to eliminate a single mode (n,ℓ). In this case, Eqs. (39) and (40) become

$$\frac{A_{k\nu^*}}{Q_G} = -\frac{FP_{BG\pm}(\nu,\ell)}{FP_{W\pm}(\nu^*,\ell)\delta_{\nu\nu^*} + FP_{BW\pm}(\nu,\ell)} \tag{41}$$

and

$$E_\pm(\nu,\ell) = 0 \tag{42}$$

respectively. Thus, complete elimination of a single mode is always possible. One should note, however, that elimination of a single mode or minimization of the acoustic power of a single circumferential wave number does not necessarily result in a decrease of the total acoustic power.

III. Numerical Examples and Discussions

A. Conditions Investigated

In the following numerical examples, the geometrical parameters of the rotating cascade, the gust, and the actuator are fixed as follows:

Hub-to-tip ratio: $h = 0.4$;
Tip speed/axial velocity: $\Omega = 2.4744$;
Axial chord length/tip radius: $C_a = 0.055$;
Number of blades: $N_B = 40$;
Gust wave number: $N_G = 35$;
Actuator width: $2L = C_a$.

The axial and radial variations of the gust velocity profile are neglected (i.e., $f_{N_G}(r,z) = 1$). Therefore, the gust is assumed to be of a simple harmonic sinusoidal form:

$$q_{G,z} = -Q_G e^{iN_G\hat{\theta}} = -Q_G e^{iN_G\theta - iN_G\Omega t} \tag{43}$$

The axial mode number k of the actuator motion is fixed to $k = 1$. Therefore,

$$a(\theta, z)e^{-iN_G\Omega t} = A_{k\nu^*} \sin\{Z(z)\} e^{i(\nu^* N_B + N_G)\theta - iN_G\Omega t}$$
$$= A_{k\nu^*} \sin\{Z(z)\} e^{i(\nu^* N_B + N_G)\hat{\theta} + i\nu^* N_B \Omega t} \tag{44}$$

Further, we denote

$$A_{k\nu^*} = |A_{k\nu^*}| e^{i\Psi_{k\nu^*}} \tag{45}$$

The effects of the control heavily depend on the number of cut-on acoustic modes. Under the specification given here, the only parameter to control the number of the cut-on modes is the axial Mach number M_a. In the cases of low Mach numbers in which only a single acoustic mode is cut on, complete antisound cancellation of the cut-on acoustic mode is always possible, as shown by Eqs. (41) and (42). But it should be noticed that simultaneous elimination of upstream and downstream acoustic powers E_- and E_+ is not possible because the right-hand side of Eq. (41) gives different values for subscripts $+$ and $-$.

B. Control of Primary Sounds of Multiple Cut-On Modes

In this paper, we investigate a case of $M_a = 0.20$, in which four duct modes $(\nu, \ell) = (-1, 0), (-1, 1), (-1, 2)$, and $(-1, 3)$ are cut on. In this case, the circumferential wave number of the cut-on modes is restricted to a single number of $n = \nu N_B + N_G = -N_B + N_G = -5$. Therefore, the cut-on frequency is also restricted to a single number of $\nu N_B \Omega = -N_B \Omega$. Hereafter, the parameter ν, instead of the circumferential wave number $n(=\nu N_B + N_G)$, is used for indicating the circumferential mode.

There are many options in the choice of the mode ν^* of the actuator motion. The actuator motion with the cut-on mode number $(\nu^* = -1)$ generates the direct secondary sound waves of the cut-on mode, which propagate in up- and downstream directions without decaying and with rather short axial wavelengths. On the other hand, the direct secondary sound waves induced by the actuator motions with cut-off mode numbers $(\nu^* \neq -1)$ are composed of cut-off modes only, and exponentially decrease with the axial distance. The axial wave lengths are longer than that of the cut-on mode. Note that in the case of a negative mode number, $\nu^* = -1$ for example, (i.e., $n^* = \nu^* N_B + N_G = -5$), the actuator surface motion makes a traveling wave form spinning in the direction opposite to the rotor rotation.

The suppression of noise by the actuator motion with the cut-on mode number is, in principle, based on the concept of the antisound. But we should take notice that the primary sound $E_{BG\pm}(\nu, \ell)$ is suppressed by the sum of the direct secondary sound component $E_{W\pm}(\nu^* = \nu, \ell)$, which is generated directly from the actuator surface, and the blade-interfering secondary sound component $E_{BW\pm}(\nu, \ell)$, which is produced due to interaction of the direct secondary sound waves from the actuator with the rotor blades.

On the other hand, we should note that the actuator motion of the cut-off mode ($\nu^* = -2$, for example) can also suppress the acoustic power. The mechanism of suppression with actuator motions of cut-off modes is, however, somewhat different from the antisound concept because the direct secondary disturbances generated by the actuator surface motion are of cut-off modes and cannot directly contribute to the suppression of the existing primary sounds of cut-on modes $E_{BG\pm}(\nu, \ell)$. They are attenuated only by the cut-on component of the blade-interfering secondary disturbances $E_{BW\pm}(\nu, \ell)$ resulting from the interaction of the direct secondary disturbances with the rotor blades. Therefore, the mechanism should be called aeroacoustic control rather than acoustic control.

First of all, we should note from Eq. (41) that complete suppression of acoustic powers is no longer attainable in the case of multiple cut-on modes.

Secondly, we should notice from Fig. 2 that the actuator motion with the same circumferential mode number as that of the cut-on modes (i.e., $\nu^* = -1$) can attain only a little attenuation of the sound level. On the other hand, actuator motions with cut-off circumferential mode numbers $(\nu^* \neq -1)$ are able to realize substantial decrease of the sound level by a factor higher than 10 dB. This implies that the cancellation by the direct secondary sound from the actuator, or in other words, the concept of the antisound, is essentially ineffective for the sound waves of multiple cut-on radial modes as far as the actuator is formed as a single annular array of loudspeakers.

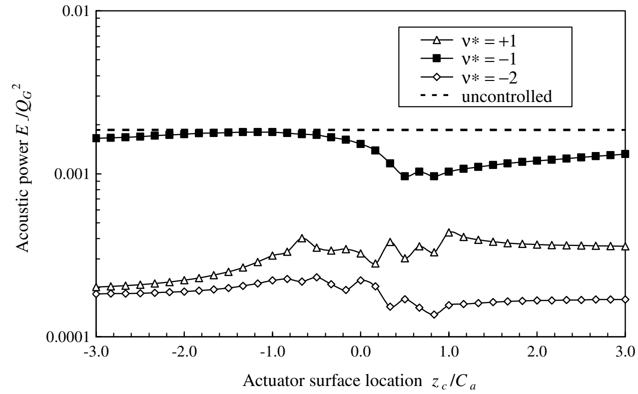

Fig. 2 Dependence of the minimized total upstream acoustic power E_- on the actuator mode ν^*.

As to the best choice of the motion, the lowest cut-off order mode $\nu^* = -2$ (nearest to the cut-on mode $\nu = -1$) spinning in the direction opposite to the rotor rotation is most advantageous because the attainable reduction of the sound level is comparable with or larger than other cases (Fig. 2), whereas the required amplitude is lowest among motions of cut-off order modes $(\nu^* \neq -1)$ as seen in Fig. 3. As to the recommended location of the actuator surface, to locate the actuator surface slightly off the rotor is most desirable, because the optimum phase of the actuator motion is insensitive to

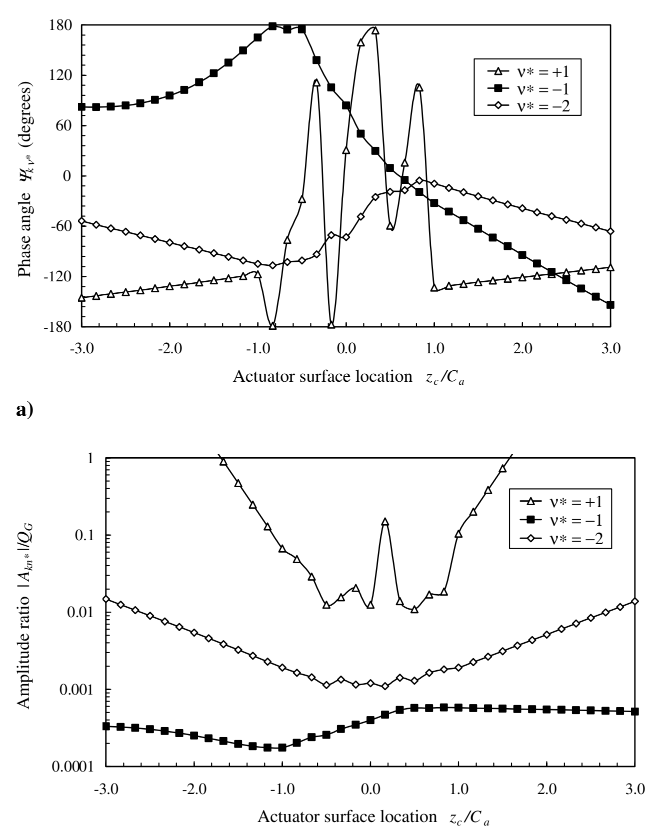

Fig. 3 Dependence of a) the optimum phase and b) the required amplitude of the actuator motion on its mode number ν^* for minimization of the total upstream acoustic power E_-.

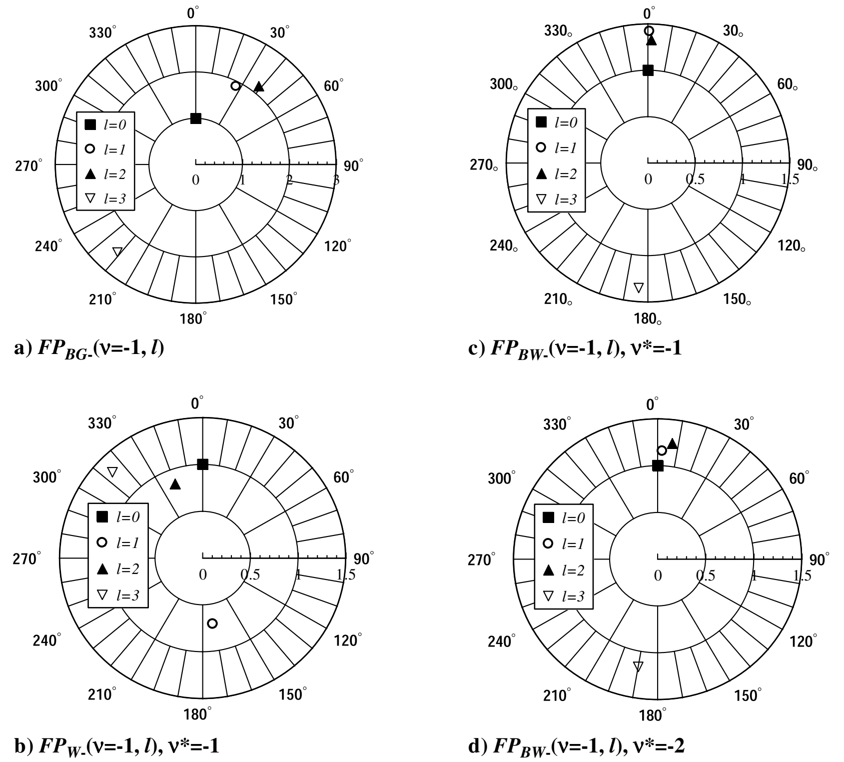

Fig. 4 The polar diagrams of the components: a) $FP_{BG-}(-1,\ell)$; b) $FP_{W-}(-1,\ell)$; c) $FP_{BW}(-1,\ell)$, produced by the actuator mode of $\nu^* = -1$; d) produced by the actuator mode of $\nu^* = -2$. The complex values are normalized by that of mode $(-1, 0)$ of each component.

change in the axial location, and the required amplitude is not too large (Fig. 3).

The reason for the crucial difference in the effects between the actuator motions of the cut-on and cut-off modes is clearly found from Fig. 4, in which relative magnitudes and phases of complex numbers of $FP_{BG-}(-1,\ell)$, $FP_{W-}(-1,\ell)$, $FP_{BW-}(-1,\ell)$ produced by the actuator mode of $\nu^* = -1$ and $FP_{BW-}(-1,\ell)$ produced by that of $\nu^* = -2$ are shown in polar diagrams. One can see that the structure (relative relationship in phase and magnitude among different radial orders) of the direct secondary sound from the actuator $FP_{W-}(-1,\ell)$ with $\nu^* = -1$ is significantly different from that of the primary gust–rotor interaction noise $FP_{BG-}(-1,\ell)$. Therefore, it is impossible to effectively cancel all radial components of $Q_G FP_{BG-}(-1,\ell)$ by all radial components of $A_{k\nu^*} FP_{W-}(-1,\ell)$ simultaneously with any value of $A_{k\nu^*}$. On the other hand, the structure of the component $FP_{BW-}(-1,\ell)$ is quite similar to that of $FP_{BG-}(-1,\ell)$, irrespective of the mode parameter ν^* of the actuator motion. Therefore, $Q_G FP_{BG-}(-1,\ell)$ can nearly be cancelled by $A_{k\nu^*} FP_{W-}(-1,\ell)$, with a suitably selected complex amplitude ratio $A_{k\nu^*}/Q_G$.

This conjecture can be confirmed by Fig. 5, in which the total upstream acoustic power E_- and its modal components $E_-(-1,\ell)$ are shown for the uncontrolled primary sound, the minimized sound by the actuator motion of cut-on mode $\nu^* = -1$, and that of the cut-off mode $\nu^* = -2$. It is obvious that control by the mode $\nu^* = -2$ successfully reduces each radial mode, whereas that by the mode $\nu^* = -1$ fails to reduce all radial modes simultaneously.

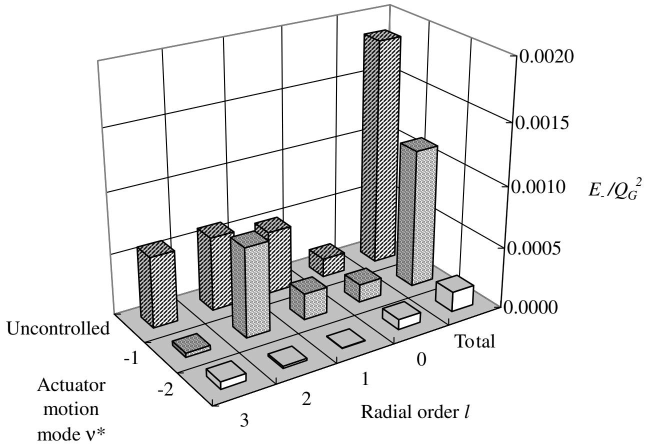

Fig. 5 Total upstream acoustic power E_- and its modal components $E_-(-1,\ell)$ of the uncontrolled primary sound and the minimized sound by the actuator motion of mode $\nu^* = -1$ and that by the actuator motion of mode $\nu^* = -2$. The location of the actuator is $z_c/C_a = -1.5$.

IV. Conclusions

The problem of suppressing the tone noise due to interaction of the rotor blades with the gust by means of the actuator motion on the duct surface is studied. Theoretical analysis with numerical calculations is conducted for a simple harmonic sinusoidal gust and a simple harmonic sinusoidal circumferential wave form of the actuator motion, and optimum conditions of the actuator motion are investigated. The results are summarized as follows:

1) If the noise field is composed of a single cut-on duct mode only, complete suppression of either upstream acoustic power or downstream acoustic power is possible.

2) Simultaneous suppression of upstream and downstream acoustic powers is in general difficult.

3) In the case of multiple cut-on duct modes, complete suppression of noise is unattainable.

4) The modal structure of the sound waves from the actuator surface on the duct wall is substantially different from those of the sound waves generated from rotor blades.

5) In the case of the primary noise of a cut-on circumferential wave number with multiple cut-on radial modes, an actuator motion generating sound waves of the cut-on circumferential wave number cannot effectively suppress the noise because the cancellation of the original gust–rotor interaction noise by the direct secondary sound from the actuator surface is essentially unsuccessful.

6) In the case of multiple cut-on radial modes, actuator motions of a cut-off circumferential wave number can successfully suppress the

sound level, provided that the actuator motion of a large amplitude is possible. In this case, the original primary noise of cut-on modes due to rotor–gust interaction is subtracted only by the blade-interfering secondary noise of cut-on radial modes due to interaction of the rotor blades with the direct secondary disturbances from the actuator, whereas the direct secondary disturbances, which are cut off, decay exponentially as they propagate upstream and downstream.

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